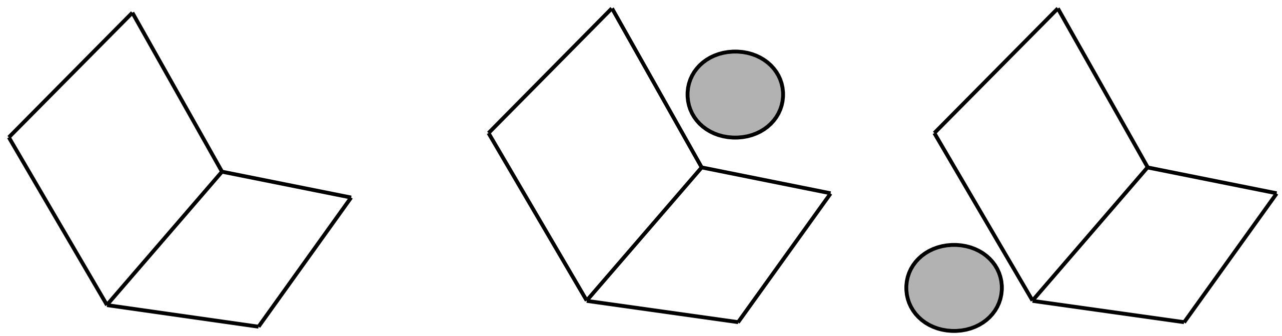
There's **something** about Math

Can you find a child who cannot use a cell phone?

While I share your concerns about education in general, and math education in particular, I'm afraid you'll find me no better informed than the average professor of college mathematics. It sounds like you are hoping for someone with more on-the-ground experience.

There's **something** about Math



What fraction of a year is a month?

What fraction of a year is a week?

What is the total weight of six 8 ounce packages of turkey?

What is the definition of a fraction?

Add times: 2:45 3:51 1:35 1:52 9:23 10:03

What is 0/0?

Guess Your Number

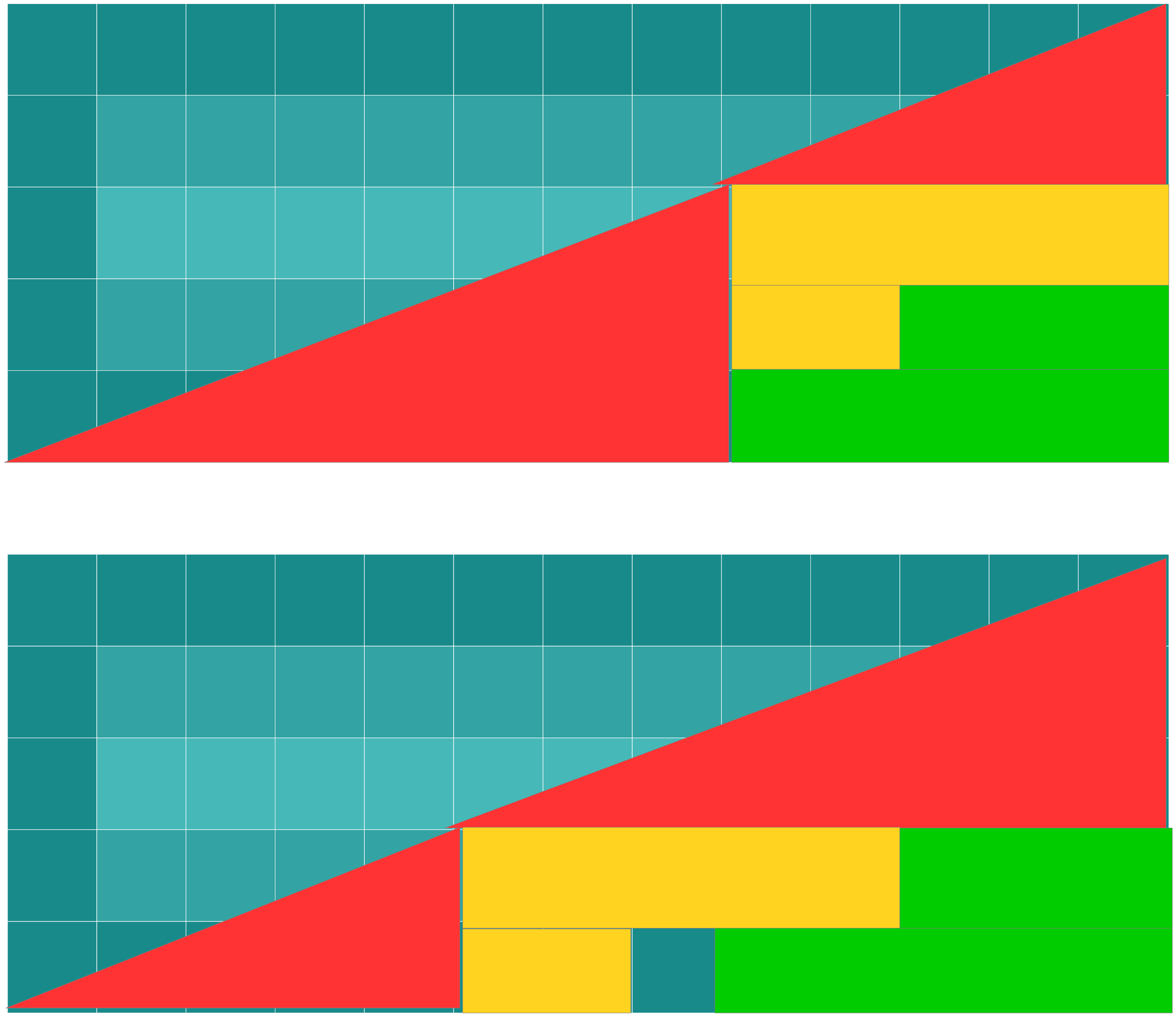
1	3	5	7
9	11	13	15

2	3	6	7
10	11	14	15

4	5	6	7
12	13	14	15

8	9	10	11
12	13	14	15

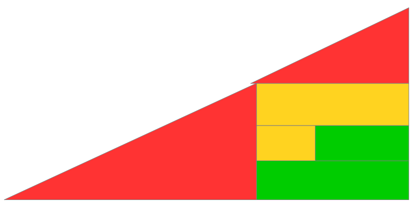
From whence came the extra square?

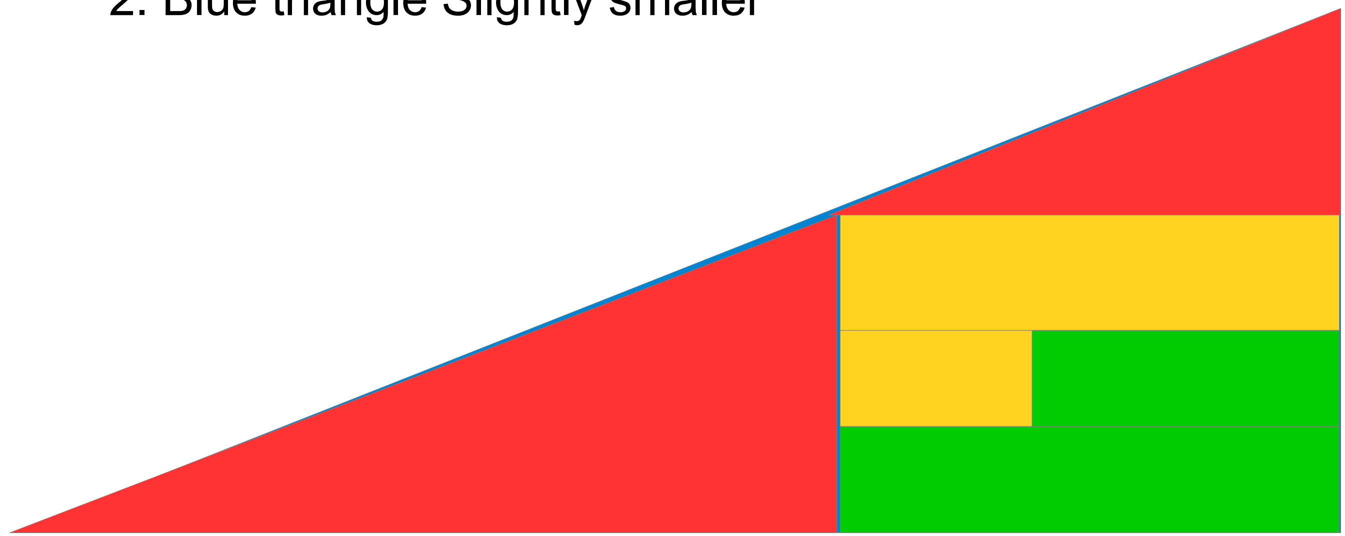


Look Closely
1. Blue triangle Slightly bigger
2. Blue triangle Slightly smaller

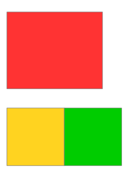
Blue Triangle

Area=.5x5x13=32.5

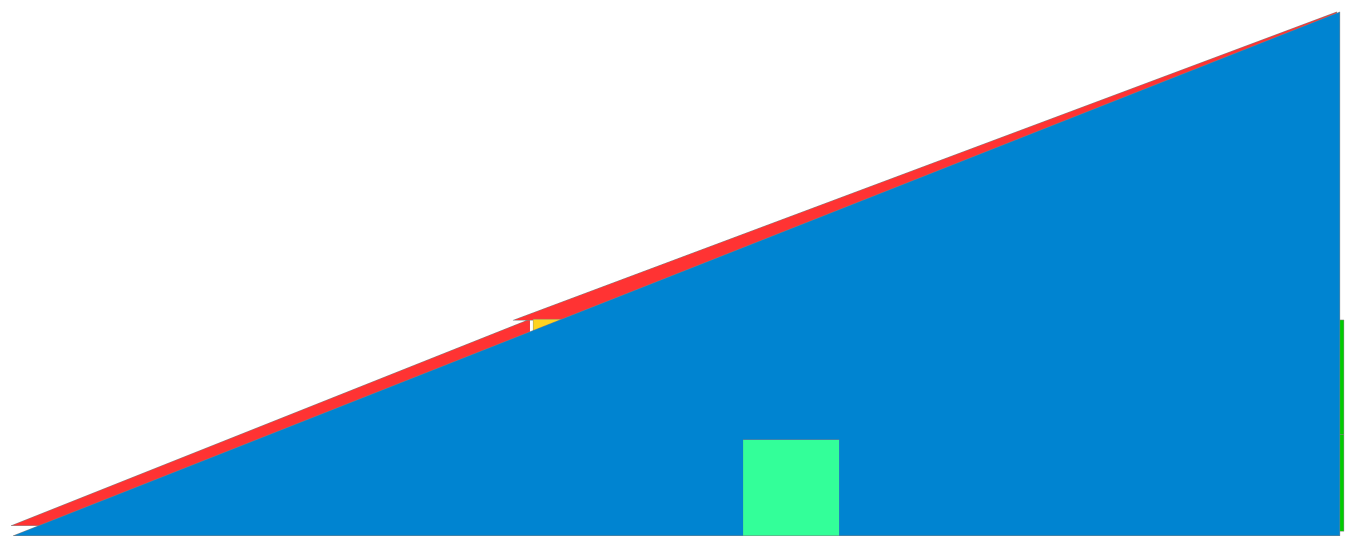




Area=
.5x2x5= 5
.5x3x8=12
3X5=15
32



32.5-32=.5



32+1-32.5=.5

Magic Squares

A	B	C	D
C	D	A	B
D	C	B	A
B	A	D	C

A	B	C	D
D	C	B	A
B	A	D	C
C	D	A	B

ABCD
ABDC
ACBD
ACDB
ADBC
ADCB

A=1 B=2 C=3 D=4 A=0 B=4 C=8 D=12

1	2	3	4		0	4	8	12		1	6	11	16
3	4	1	2	+	12	8	4	0	=	15	12	5	2
4	3	2	1		4	0	12	8		8	3	14	9
2	1	4	3		8	12	0	4		10	13	4	7

A=0 B=4 C=8 D=12 A=1 B=2 C=3 D=4

0	4	8	12		1	2	3	4		1	6	11	16
8	12	0	4	+	4	3	2	1	=	12	15	2	5
12	8	4	0		2	1	4	3		14	9	8	3
4	0	12	8		3	4	1	2		7	4	13	10

Zero and one are identity elements

$0+a=a$ $1 \times a=a$

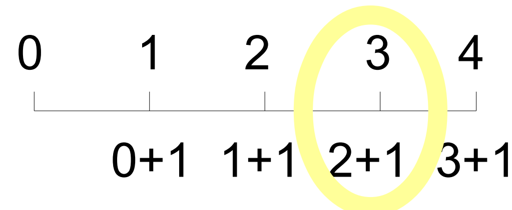
$0 \times a=0$ $1^a=1$

There is a problem when a = ∞.

Is 1 nothing, because when we multiply it by a number we get that number. We resolve the problem by identifying **zero** as the **identity element** for addition and **one** as the **identity element** for multiplication.

Zero is the number before 1

An argument for zero being nothing is that it is a place holder. So what, every number is a place holder. Math is a discovery and validation process.



We create our numbers by adding1 to the previous number.

0+1=1	observed
3=1+2	commutative property (turn around facts)
3=(0+1)+2	substitute 0+1 for 1
3=0+(1+2)	associative property (add 1+2 first rather than 0+1)
3=0+3	substitute 3 for 1+2

Not a rigorous proof but it shows us that when we add 0 to a number we get that number as the answer.

Yes, zero is a number.

Discovering and naming the numbers to the left of zero

B	**A**	**0**	**1**	**2**
	B+1	**A+1**	**0+1**	**1+1**

Using the same technique for discovering negative numbers.

$A+1=0$

$A=0-1$ subtraction is inverse addition (2+3=5 then 2=5-3)

$A=-1$ **notation**

since $A+1=0$

$-1+1=0$ substitution

This is the definition of a negative number: **When a negative number is added to its inverse, we get zero.**

Further validation

$B+1=A$ how we create our numbers

$B+1+1=A+1=0$ add 1

$B+2=0$ association and substitution

$B=0-2$ subtraction

$B=-2$ notation

$B+2=0$

$-2+2=0$ substitution

We could now use the definition of negative numbers to prove the four rules of operstion rather then blindlessly accepting them.

Place holder

4.4 = 04.4 =4.40 no affect
40.4>4.4>4.04 bigger and smaller
41.4>4.4<4.14

4.1 | -1| 4=410-10+4=404

Not needed to build numbers
0 1 2 3 4 5 6 7 8 9 11 12 13 14 15 16 17 18 19 21 base 9 2x9+1=19
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 base 10

Interpretation
0+a=a 1xa=a identity element (not zero=nothing)
0xa=0 1^a=1

Sign of zero
A 0 1 2 3 4
A+1 0+1 1+1 2+1 3+1 building numbers

2=1+1 =(0+1)+1=0+(1+1)=0+2

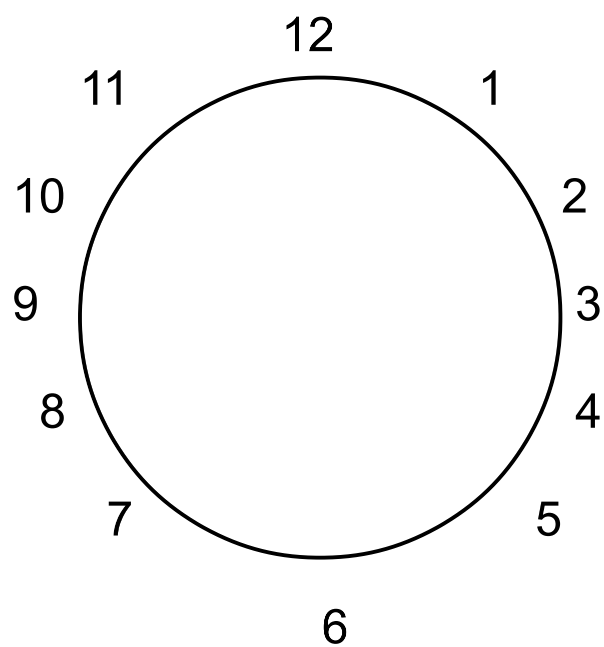
A+1=0 0+0=0
A=0-1=-1 notation 0=0-0=-0
-1+1=0 defintion neg. no.
1=0 - -1 rule for subtracting neg nos.

More discovery

0+0=0	zero added to a number(zero)
0=0-0	subtraction
0 = -0	**notation**.

Zero has a sign but 0 and -0 occupy the same place on the number line.
While they may be equivalent, they are not necessarily the same.

Another approach to illustrating zero is nothing as misleading



12+1= 12	0+1=1
12x5=12	0x5=0
12-1=11	0-1=-1
5+11=5-1=4	5+-1=4
11x11=121=1	-1x-1=1

At 12: -24, -12 0 12 24
At 11: -13 -1 11 23 35

Making decisions

$3^0=1$ $0^3=0$ $(-0)^3=-0$

$2^0=1$ $0^2=0$ $(-0)^2=0$

$1^0=1$ $0^1=0$

$0^0=1$? **$0^0=0$**

$-1^0=1$ $0^{-1}=\infty$

$.1^0=1$ $0^{.1}=0$

$-.1^0=1$ $0^{-.1}=\infty$

$0/0=0 \times 1/0=0 \times \infty$

0 divide by **anything** is 0

0 multiplied by **anything** is 0

Anything divided by itself is 1

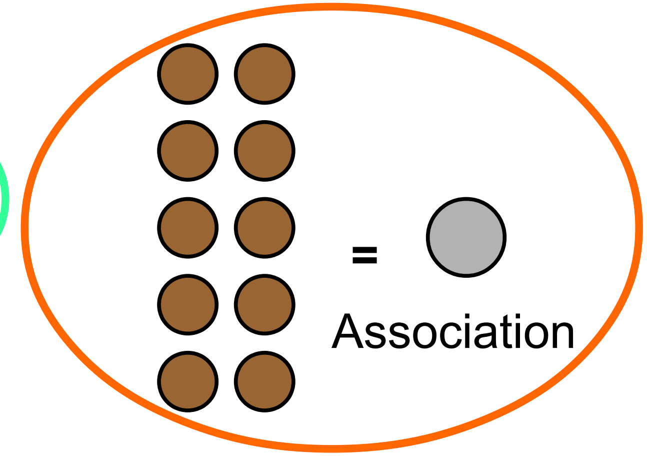
$0^0=1$ $0^{-0}=\infty$

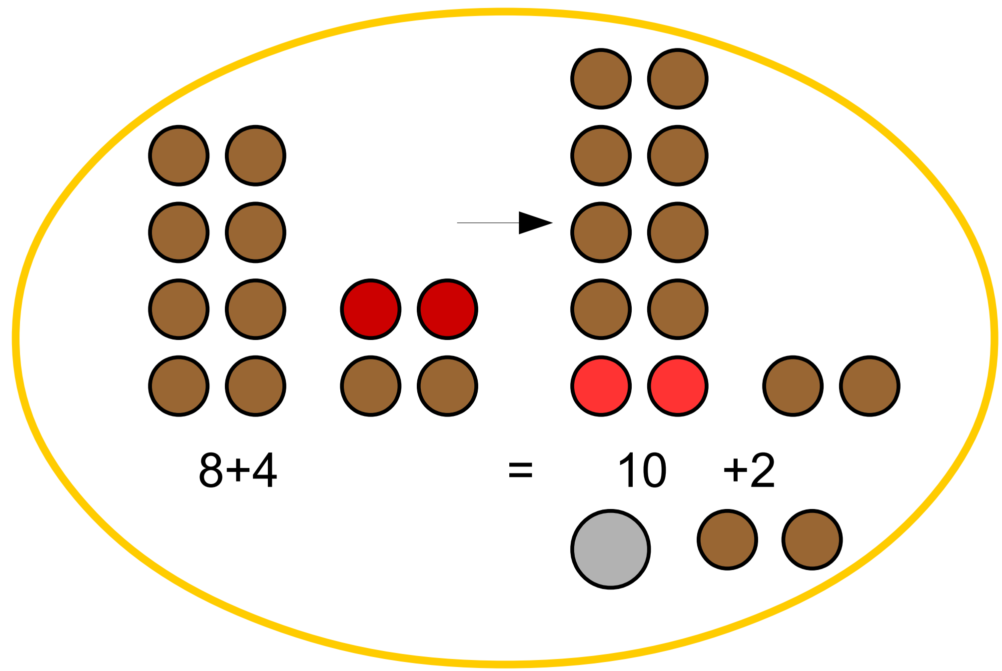
There is more!

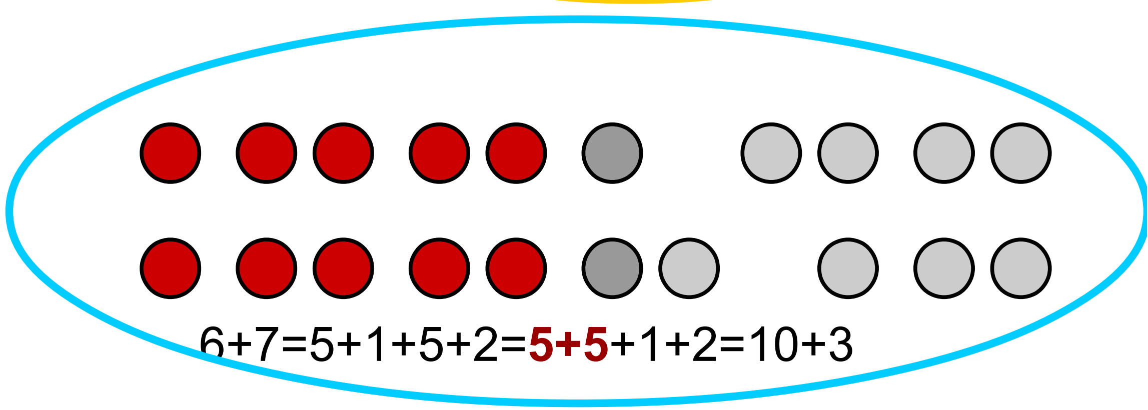
Addition

+	1	2	3	4	5	6	7	8	9
1	2								
2	3	4							
3	4	5	6						
4	5	6	7	8					
5	6	7	8	9	10				
6	7	8	9	10	11	12			
7	8	9	10	11	12	13	14		
8	9	10	11	12	13	14	15	16	
9	10	11	12	13	14	15	16	17	18

1	2	3	4	5
9	8	7	6	5
10	10	10	10	10







Making Tens

Nine rule

Add ten count back 1

Eight rule

Add ten count back 2

Nickel rule

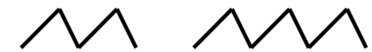
7=5+2, 6=5+1, 4=5-1, 3=5-2

Others

6+3=10-1, 7+4=10+1,
4+3=7 4+4=7+1 3+3=7-1

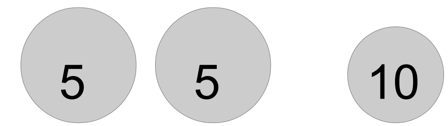
Multiplication

x	1	2	3	4	5	6	7	8	9
1	1								
2	2	4							
3	3	6	9						
4	4	8	12	8					
5	5	10	15	20	25				
6	6	12	18	24	30	36			
7	7	14	21	28	35	42	49		
8	8	16	24	32	40	48	56	64	
9	9	18	27	36	45	54	63	72	81



2x3=2+2+2 =0+2+2+2

Definition of Multiplication



2 nickels= 1 dime

1	2	3	4	5
9	8	7	6	5
10	10	10	10	10

Double Trouble

4x7=(2x2)x7=2x(2x7)=(2x7)+(2x7)=14+14=28

Half a loaf is better than nothing

8x5=(4x2)x5=4x(2x5)=4x10=40

Half a loaf followed by a loaf

8x6=8x(5+1)=8x5+8x1=40+8=48

One less followed by takeaway from 10

8x9=8x(10-1)=(7+1)x10-8=7x10+(10-8)

Distributive property
8x(5+1)=8x5+8x1

Associative property
(2x2)x7=2x(2x7)

Commutative Property
8x5=5x8

Is zero a place holder?

We can construct our numbers without using a zero, even though zero does exist in that number system.

In base 9; 121= 1x9^2+2x9^1+1=81+18+1=100_{10}

However, we find it more difficult to perform, long addition and multiplication. If we use the zero, we get an ambiguous representation of number: 20=2x9+0=18 19=1x9+9=18

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	base 10
1	2	3	4	5	6	7	8	9	11	12	13	14	15	16	17	18	19	21	22	base 9

+	1	2	3	4	5	6	7	8	9
1	2								
2	3	4							
3	4	5	6						
4	5	6	7	8					
5	6	7	8	9	11				
6	7	8	9	11	12	13			
7	8	9	11	12	13	14	15		
8	9	11	12	13	14	15	16	17	
9	11	12	13	14	15	16	17	18	19

x	1	2	3	4	5	6	7	8	9
1	1								
2	2	4							
3	3	6	9						
4	4	8	13	17					
5	5	11	16	22	27				
6	6	13	19	26	33	39			
7	7	15	23	31	38	46	54		
8	8	17	26	35	44	53	62	71	
9	9	19	29	39	49	59	69	79	89

We need and addition and multiplication table for computations

$.1_{10}$=.080808...$_9$=.1|-1|1|-1|1|-1.....$_9$

4.1|-1|4=410-10+4=404

By the way, any number can be a placeholder

Some Computations

1.9=1+9/9=2 1.9 x9= 18.9=19
1.99=1+ 9/9+ 9/9x9= 1+ 1 + 1/9=2.1
1.89=1+8/9 + 9/9x9=1+8/9+/1/9=2

			Check work	
123_9	81+18+3=	102	5656	
45_9	36+5 =	x 41		
626		102	5x729 =	3645
493		408	6x81 =	486
5656_9		4182	5x9 =	45
Notice zero was not used			6x1 =	6
				4182

Patterns
Redefining multiplication and exponentiation

We must know rules for adding and subtracting negative numbers

	3X2=6			3x-2=-6	
3-1=1	2x2=4	4-6=-2	2-3=-1	2x-2=-4	-4 - -6=2
2-1=1	1x2=2	2-4=-2	1-2=-1	1x-2=-2	-2 - - 4=2
	0X2=0			0x-2=0	
	-1x2=-2			-1x-2=2	
	-2x2=-4			-2x-2=4	
				-3x-2=6	

Definition of multiplication				Definition Of exponentiation	
3X3=3+3+3		**3x3=0+3+3+3**		3^3=3x3x3	**3^3=1x3x3x3**
3X2=3+3	drop +3	3x2=0+3+3	drop +3	3^2=3x3	3^2=1x3x3
3X1=3	drop 3	3x1=0+3	drop +3	3^1=3	3^1=1x3
3X0=0		3x1=0		3^0=0	3^0=3

3^0=3^1-1=3/3=1 validation

3x2 0 1 2 3 4 5 6 0 1 2 3 4 5 6

3X2=2+2+2 3x2=0+2+2+2

0x2=0+0x2
0x2-0x2=0
0x(2-2)=0

0x2+-2x0=0
-2x0=0-0x2
-2x0=0-0
-2x0=-0 but 0=-0

-3x0=0
-3x(2+-2)=0
-3x2+-3x-2=0
-3x-2=0-(-3x2)=3x2

Different ways to multiply
Distributive property

```
  4          4    4
 18         18    8       20-2      17 -1      18x16=2x2x8x9=4x72=288
x16        x16   +6      x   16    x17+1
108        288   18       320       289
18               10       -32        -1
288              28       288       288
```

Very Long Multgtiplication

For the final example, we want to illustrate dramatically, why the current approach to teaching long multiplication has to be modified.

12	120 +3		1230+4	
x12	x120+3		x1230+4	
144	14400	12x12x100	1512900	making use of previous work
	360	12x3x10	4920	
	360	12x3x10	4920	copied line above
	9	3x3	16	
	15129	we are using the distributive property	1522756	

12340+5	123450+6
x12340+5	x123450+6
152275600	15239902500
61700	740700
61700	740700
25	36
152399025	15241383936

123456123456123456	123456x(10^12 +10^6+1)
x123456123456123456	
15241383936	123456x123456x1
15241383936	123456x123456x10^6
15241383936	123456x123456x10^12
15241399177399177383936	123456x123456123456123456
15241399177399177383936	123456x10^6x123456123456123456
15241399177399177383936	123456x10^6x123456123456123456
1524141441881359618229078311338**3936**	**This is showing your work**

Zero is a not a place holder base 10

404=4|0|4 = **4.1|-1|4**=4.1x100-1x10+4=410-10+4=404

.001=**1|-9|-9**=1/10 -9/10 -9/100 = (100-90-9)/1000=1/1000=.001

```
1|-9|-9
     x2
    -1 8
  -1 8
  2
1|-9|-8=(100-90-8)/1000=2/1000=.002
```

Yes, we can avoid using zero as a place holder. Fortunately, we used negative numbers and fractions to achieve this goal. **We use the zero because it makes It easier to perform calculations and to order our numbers.**

404= 4.1|-1|4= 4.2|-2|4 Is the third number the biggest?

Using a different base (base 3)

0	1	2	3	4	5	6	7	8	9	10
000	00**1**	0 1 **-1**	0 1 **0**	0 1 **1**	1 -1 **-1**	1-1 **0**	1 -1 **1**	1 0 **-1**	1 0 **0**	1 0 1

Zero leads 1 leads

An overwhelming problem showing that 1/7x1/7=1/49 in decimals
(Superiority of man over machine)

142857142857142857142857142857142857142857142857
x142857142857142857142857142857142857142857142857
20408122449
20408122449
20408142857122449
20408142857122449
20408163265265306122449
20408163265265306122449
20408163265285714285714265306122449
20408163265285714285714265306122449
20408163265306122448979551020408163265306122449
20408163265306122448979551020408163265306122449
20408163265306122448979571428571428571428571428551020408163265306122449
71428571428571428571428551020408163265306122449
91836734693877551020408122448979591836734693877551020408163265306122449
204081632653061224489795

1/49=0.02040816326530612244897959183673 best we can do with calculator

Standard 2304
other 512 save 56 digits or 1792
Binary 456

Rules for Fractions

1=1/3 + 1/3 +/13 = 3 x 1/3 definition of a fraction
1/3 = 1 / 3

A fraction results from a division

			Identity
2x1=2	identity	3x1=3	2x1=2
2x1/3= 2 / 3	division	3x1/3 =3	1=2/2, and
2x(1/3) 2 /3	association	3x 1/3 = 3/3 =1	2=2/1

Rule 1

(3 x 1/3) x (5 x 1/5)=1x1=1 defintion
3X5 x 1/3 x 1/5 =1 commutative
(3x5) x (1/3 x 1/5) =1 association
1/3 x 1/5 = 1 / (3x5) division
1/3 x 1/5 = 1/(3x5) transitive

When multiplying fractions, multiply denominators

Rule 2

(3 x 1/3) x (5 x 1/5)=1x1=1
3x 1/5 x 5x 1/3) =1
(3/5) x (5/3) =1
3/5=1 / 5/3

When dividing fractions invert and multiply

Rule 3

1/7+2/7+3/7=(1+2+3) x 1/7 = 6 x 1/7 =6/7
Distributive property

When adding fractions with like denominators, add te numerators.

Rules for Decimals

23.45= 2x10^1 + 3x10^0+ 3/10^1 + 3/10^2

Adding Decimals
.4=4/10 .05= 5/100 4/10+5/100=4/10 x10/10 +5/100=40/100+5/100=45/100

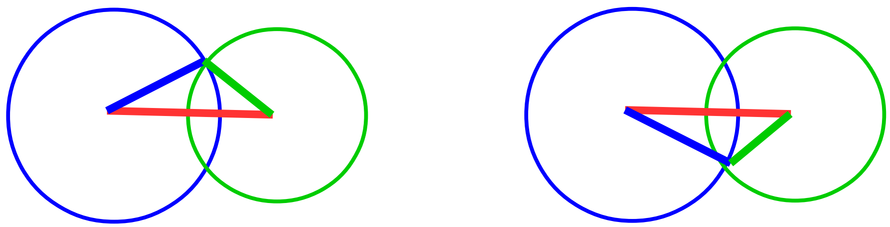
Multiplying decimals
22.5x12.43=22+5/10 x 12+43/100 =22+5/10 x 12+43/100=225/10 x1243/100
=225x1243/100

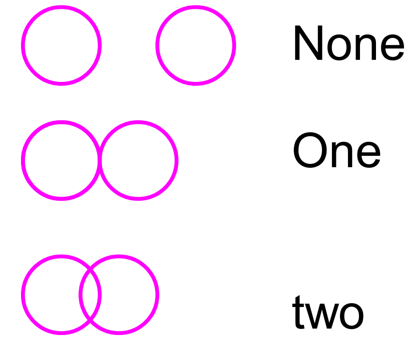
```
  22.5
  12.43
   675
  900
 450
225
279.675
```

Percents
100%=1 %=1/100
25%=25 x 1/100=.25=.25x1=.25x100%==25%

Congruent Triangles

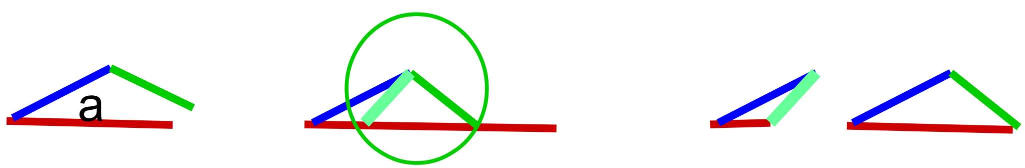
Three sides (SSS)



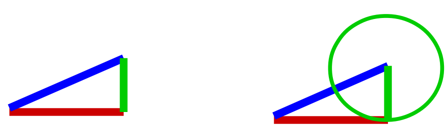


When we copy the sides we get two possible triangles (circles intersect twice). We cut the triangles out and put them on top of each other. They coincide (the second one had to be flipped)

Two sides and an angle

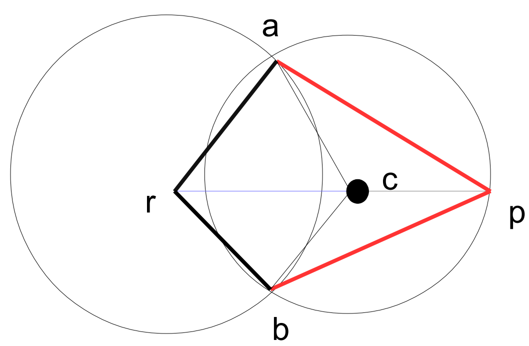


When we copy, we get two triangles which obviously will not coincide. There is one case in which we will.



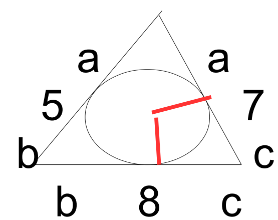
Conguency statements are postulates because they are based on observation and generalization—not by proof.

Inscribing a circle in a triangle

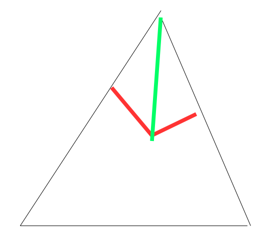


ra=rb radii circle r
ca=cb radii circle c
rc=rc self
Δrab≈ Δrbc SSS

rp=rp self
arc= brc congruent triangles
ra=rb raddi circle r
Δpra≈Δprb SAS
pa=pb sides of congruent triangle



a+b=5
a+c=7
b+c=8
(a+b)+(a+c)-(b+c)=5+7-8=4
2a=4 a=2, b=3 c=5

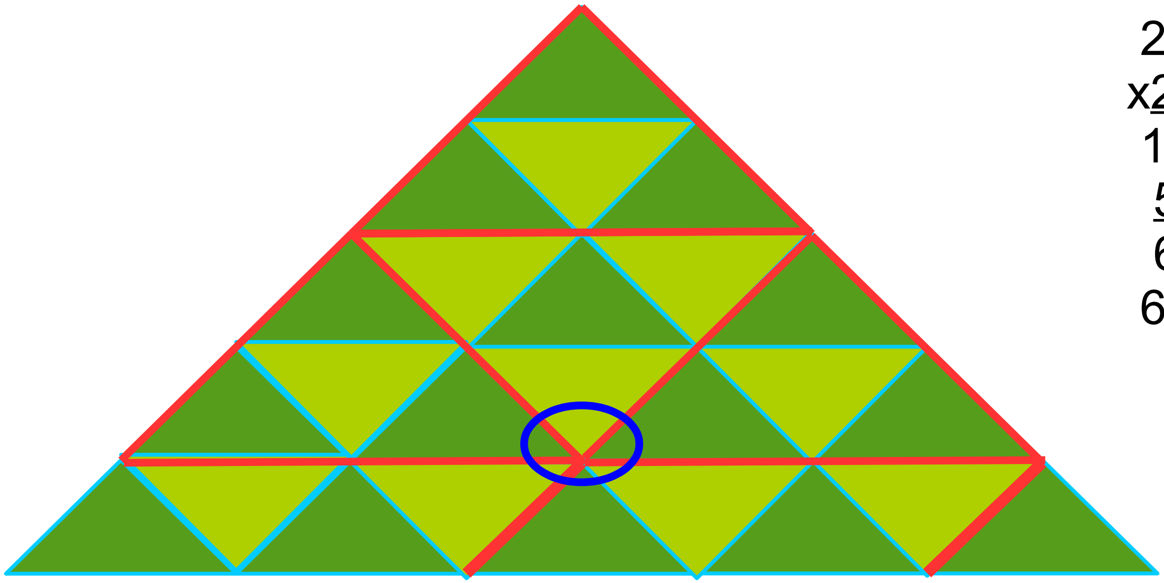


green line bisects angle
red line rt. angle
AAS making red lines equal
and other side

We do not have to bisect angles, and construct perpendiculars.

It would be difficult to bisect angles and construct perpendiculars,
but the proof gives us insight into an easier way to draw an inscribed circle.

Bringing it Altogether



1+1+1+1 + 4 x 1/4 4 x 1/4 +1/4 =6 1/4

```
 2.5
x2.5
 125
 50
 6.25=
6 25/100=6 1/4
```

```
  2 + 1/2
 x2+ 1/2
   2/2+1/2 x 1/2
4 +2/2
4+1+1+1/4=6 1/4
```

Topics

Straight lines
Similar triangles
Decimals
Fractions
Distributive property

Sides of red twice as long as sides of reen triangles
Area of green triangle ¼ that of the red triangle.

Sides of big triangle are 2.5 size of red triangles
There are 1+3+5+7+9=25 green triangles
Area of big triangle= 25/4=6 ¼ the are of the red triangles

Combining what we learned

$ax^2+bx+c=0$
$ax^2+bx+0+c*1=0$
$x^2+(b/a)x+0+(c/a)*1=0$
$x^2+(b/a)x+(b/2a)^2-(b/2a)^2+(4a/4a)xc/a=0$
$(x+b/2a)^2=b^2/4a^2-4ac/4a^2$
$x+b/2a=\pm (b^2-4ac)^{1/2})/2a$
$x=(-b\pm (b^2-4ac)^{1/2})/2a$

$e^{ix}=\cos(x)+i\sin(x)$
$e^{i\pi}=\cos(\pi)+i\sin(\pi)=-1$

$25^x+5^x=20$
$(5^x)\hat{}+5^x=20$
$y=5^x$
$y^2+y-20=0$
$Y=(-1\pm(1+80)^{1/2}/2$
$=(-1\pm 9)/2=4,-5$

$y=5^x=4$
$x\log(5)=\log(4)$
$x=2\log(2)/\log(5)$
$=2*.301/.699$
$=$**$.861$**
$25^{.861}+5^{.861}=$
$15.98+3.99=19.97$

$y=5^x=-5$
$x\ln(5)=i\pi+\ln(5)$
$x=1+i\pi/\ln(5)$
$5^{(1+i\pi/\ln(5)}=5xe^{(\ln(5)i\pi/\ln5)}$
$=5xe^{i\pi}=-5$
$-5x-5+-5=22-5=20$

The Pyramid

Ponzi Scheme

people	you	next	next	last
1				
3	**3**			
9	**9**	3		
27	**27**	9	3	
81	**81**	27	9	
121	**$120**	39-1=$38	12-2=$10	-$4

You contact 3, they contact 9, they contact 27, etc.
Each person sends a dollar back to the last 4 senders.
You receive $3 from the first level, $9 from the next,etc,
Receiving from four levels, you get $120.
The last level cannot find anymore people.
Your net out each level is what you receive minus the amount
You sent uo the pyramid,

Check that the sum of all the money received is equal to that sent.

Borrowing money at 10%

Pay $110 in onw payment at end of 10 months

			Missed Payment(fee $0)		Missed payment (fee $2)		
	Paid	Owed	Paid	Owed	Paid	owed	
		100		100		100	borrowed
1	11	90	11.00	90	11.00	90	
2	10.90	80	0	90	0	90	
3	10.80	70	21.80	70	23.80	70	
4	10.70	60	0	70	0	70	
5	10.60	50	21.40	50	23.40	50	
6	10.50	40	0	50	0	50	
7	10.40	30	21.00	30	24.00	30	
8	10.30	20	0	30	0	30	
9	10.20	10	20.60	10	22.60	10	
10	10.10	0	10.10	0	10.10	0	
	104.50		105.90		112.90		
					142.90 if he paid $10 penalty		

We did not charge interest on interest for late penalty.
In the last case you could pay nearly 43% because of a management fee.