Foundations of Mathemtics

Vector Model makes it Easy

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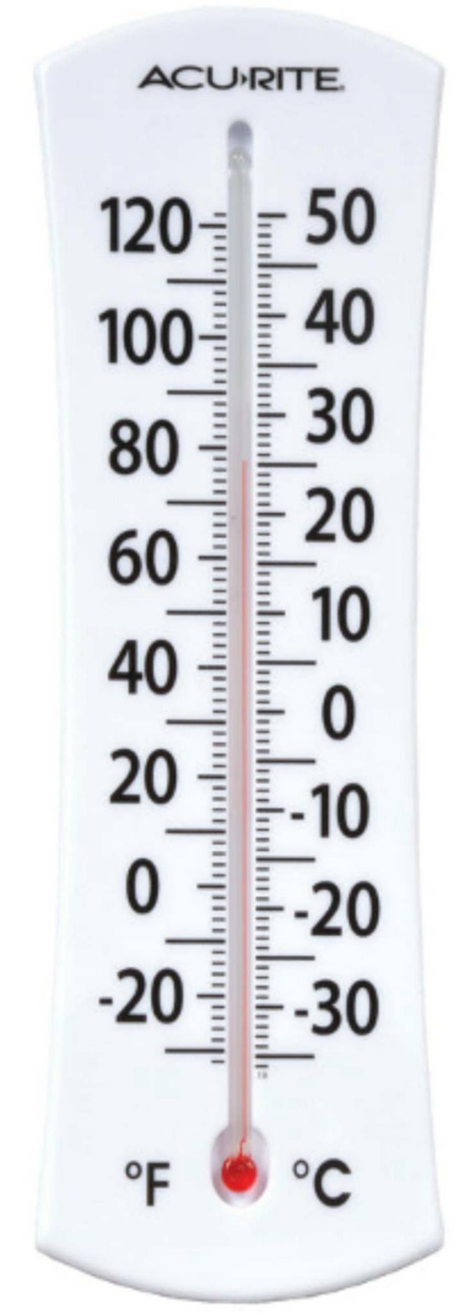
Mathematics Reinterpreted

Abstract

Is math is founded upon interpretations that are not rigorously challenged? There are many misconceptions and anomalies that can be explained by more rigorous interpretations.

Zero being considered nothing, negative numbers not be counting numbers, addition and multiplication being considered commutative, a line not having sides, an area not being negative are just a few examples of interpretations that should be reconsidered.

However, thinking of a number as a vector, one can invalidate all of these axioms. We will show the thought processes in challenging these interpretations and the removal of some anomalies resulting with existing interpretations.



Organization

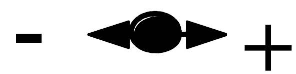
Among some of the early motivations for questioning the foundations of mathematics was trying to understand how factoring of the infinite series for trigonometric functions could be used to determine the value of pi. We will discuss this problem later in more detail. We can organize mathematics to make it easier to understand and remember:

	addition	**multiplication**	**exponentiation**
Inverses	**subtraction**	**division**	**logarithms**
geometry	**translation**	**scaling**	**rotation**
numbers	**negative nos.**	**fractions**	**irrational nos.**
spec nos.	**0**	**1**	**$\sqrt{-1} = i$**
standard	**point**	**length**	**angle**
vector	**position**	**magnitude**	**direction**

The strategy is to discuss a different way to talk about axioms to be challenge and then to show the inconsistencies in those axioms.

Zero

Zero is the start of the positive (+0) and negative numbers (-0), has no magnitude, and is represented by a dot with vectors going in the opposite directions. This interpretation implies that –0 and +0 occupy the same position since zero has no magnitude. This becomes the definition of a 'point' in geometry.

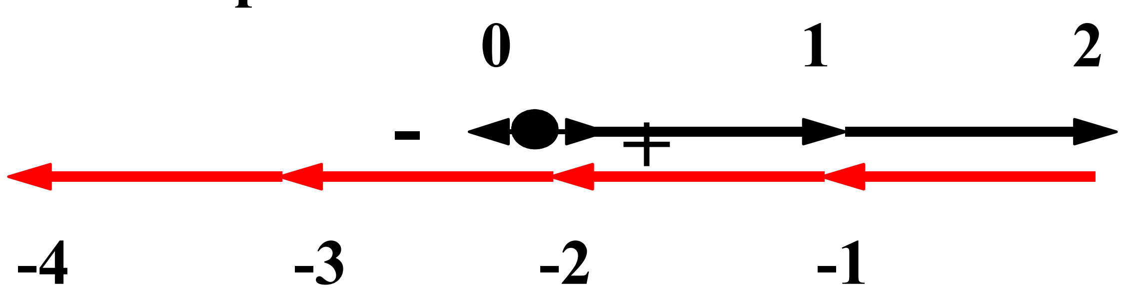


We use the function y=1/x when x=0 as illustration of this interpretation since at –0, we have -∞ and at +0, we have +∞. In computer printouts, a number very close to zero is written as –0 on the left side and +0 on the right side of zero.

According to a Google search, the statement that "0 is nothing" is a philosophical interpretation, while the existence and properties of zero are established within mathematics through a system of axioms. Zero is a core concept that is fundamental to modern mathematics. Yes, zero often represents "nothing" as the absence of quantity (like zero apples), but it's also a vital number in math, acting as a placeholder (in 205), an additive identity (x+0=x).

Negative numbers

A negative number is represented by a vector pointing in the opposite direction of a positive number.



We can see that 2+ -4 = -2 and that 2+ -2=0. From this we can derive the definition of a negative number and th rules of operation: The definition is as follows: When a negative number is added to its positive inverse, we get the identity element for addition. A negative sign is an indication of direction. Thus a negative sign changes the direction of a positive number to a negative number and a negative number to a positive number.,

Negative Numbers (continued)

This concept is a interpretation so later we will produce examples with multiplication to illustrate that it is consistent with our current knowledge of mathematics. Let us look some proofs:

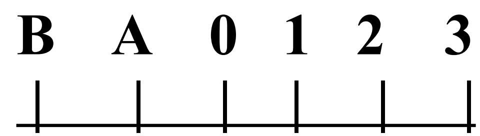
-2+2=0 definition
2= 0 – -2 subtraction confirms interpretation of - sign
-2= 0 –2 subtract explains notation of a negative number

We can see that 0+0=0 which implies that 0=0-0
We note from above that **-2=0-2**
Replacing 2 with zero we have: -0=0-0

We can see how we are lead to believe that 0 and –0 occupy the same place on the number line. We have started with negative numbers to illustrate properties of zero.

Negative Numbers (continued)

Let us start with 0. An integer is created by adding 1 to the previous number. Lets look at our number line:



		$B+1=A$	create A
		$B+1+1=A+1$	add one
*	**$A+1=0$**	**$B+2 = 0$**	substitute 0 for A+1
	$A = 0-1$	$B = 0-2$	subtract
	$A = -1$	$B = -2$	notation
	$-1+1=0$	**$-2+2=0$**	substituting for A and B

These exercises give us more confidence in our new interpretation of negative numbers. We also see that it makes it easier to do computations.

Counting Numbers

We are going to see how this new way of interpretation, allows to make math easier to learn and apply our knowledge. Traditionally, counting numbers are only positive integers greater than zero. Yet, zero and negative number can be interpreted to be counting numbers. Since multiplication is a counting process, let us look at the impact of negative numbers on multiplication.

The expression **3x2=0+2+2+2,** and **3x-2=0-2+-2+-2=0-(2+2+2)=0-6=-6** When we add repeated positive numbers, we add then a place a plus sign in front of the result. When we add repeated negative numbers, we add them as positive numbers and place a minus sign in front of the results.

When we use a negative number as a counter, we add the numbers it the traditional way, but we place a minus sign in front of the result. A negative result becomes positive and a positive result becomes negative:

-3 x –2 = 0-(-2+-2+-2) = 0 - -(2+2+2) = 0+(2+2+2) = 6
-3x2 = 0- +(2+2+2) = 0-(2+2+2) =0-6 =-6

This makes sense with the vector interpretation of numbers, but is difficult to understand from our traditional view of negative numbers. Note that in one step, you could prove that the product of negative numbers is a positive number.

Multiplication

The major impact of the reinterpretation of zero and negative numbers is upon multiplication. The convention definition of multiplication is **3x2=2+2+2**. We are going to define it as: **3x2=0+2+2+2**. This is done by pattern recognition:

3x2=2+2+2	**3x2=0+2+2+2**
2x2=2+2	**2x2=0+2+2**
1x2=2	**1x2=0+2**
0x2=?	**0x2=2**

In the first example we keep dropping a +2 but in the last step **there is no** +2 to drop.

We know that 2x0=0+0=0 or 2x0=0+0+0=0. Thus we have shown that multiplication is commutative with respect to zero without applying the commutative property.

Let us look at negative numbers **3x-2=0+-2+-2+-2=0-(2+2+2)= -6** and
-2x3=0+ -(3+3)=0-6= -6

Multiplication (continued)

Let us look at negative numbers 3x-2= 0+-2+-2+-2= 0-(2+2+2)= -6 and
-2x3= 0+ -(3+3) = 0-6 = -6

We just used our rules for multiplying by negative numbers. Thus again, we do not have to use the commutative rule to show that the multiplication with negative numbers is commutative.

However, we still note that multiplication appears to be commutative. Now let us look at the distributive property.

5x1=1+1+1+1+1=5

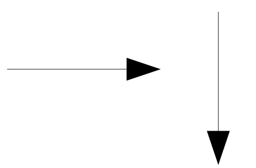
5x3=5x(1+1+1)=5x1+5x1+5x1=5+5+5=3x5

It is the distributive property that makes multiplication appear to be commutative. Let us look at the following example:

(-4)^1= (-4)^2x1/2 = ((-4)^2)^1/2 =16^1/2=4 more than 1 answer
(-4)^1= (-4)^1/2x2 = ((-4)^1/2)^2 =(2i)^2=-4

Multiplication (continued)

Thus multiplication is not commutative for fractions. Let us look at matrix multiplication:



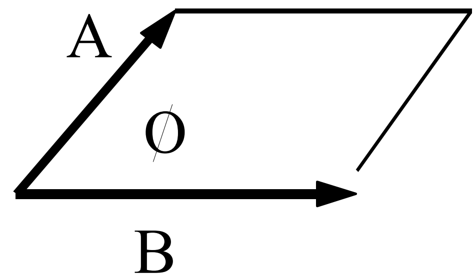
$$\begin{pmatrix}\mathbf{1} & \mathbf{2}\\ 3 & 4\end{pmatrix} \text{ x } \begin{pmatrix}\mathbf{5} & 6\\ 7 & 8\end{pmatrix} = \begin{pmatrix}\mathbf{19} & \mathbf{22}\\ \mathbf{43} & \mathbf{50}\end{pmatrix} \qquad \begin{pmatrix}5 & 6\\ 7 & 8\end{pmatrix} \text{ x } \begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} = \begin{pmatrix}\mathbf{23} & \mathbf{34}\\ \mathbf{31} & \mathbf{46}\end{pmatrix}$$

It is obvious that multiplication is not commutative for matrices. To resolve this anomaly, we make the following definitions. **Multiplication obeys the distributive property. Multiplication is not commutative.**

Thus, this subtle change in the definition of zero and negative numbers removes many of the anomalies created by the former interpretations.

Numbers are vectors

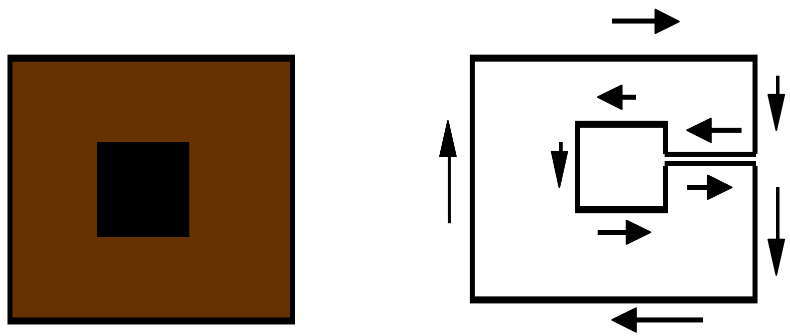
As we look at more of the anomalies in the current mathematical logic, we begin to realize, that if we think of numbers as vectors, this anomalies disappear as they did with the concept of the minus sign as an indicator of direction rather than magnitude.



The product of vectors A and B is AB sin(Φ) which is the magnitude or area of the parallelogram. If A=B and $\Phi = 90^0$ we have the area of a unit square. **Thus traditional multiplication assumes orthognality of the width and length**.

Numbers are vectors (continued)

The following diagram illustrates these points:



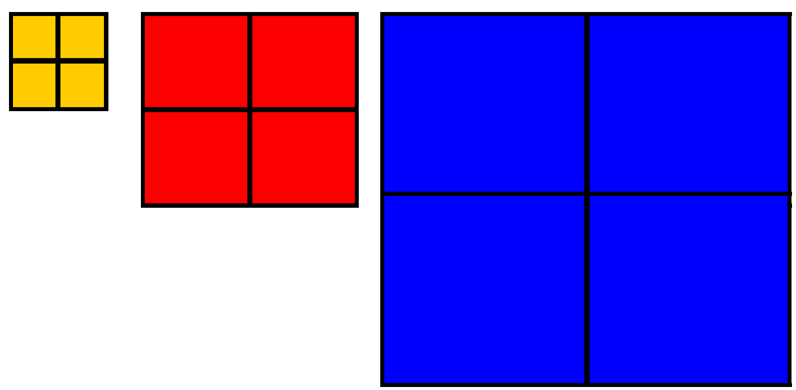
For the outer rectangle, we go clockwise and the inner rectangle we go counter clockwise. The inside of the outer rectangle is the right side of the lines. Thus the clock direction can be used to determine if an area is positive or negative. You should begin to notice these consistencies with the vector form of a number.

Fractions

Fractions are defined similary to the way negative numbers are defined. When a fraction is multiplied by its multiplicative inverse, the result is the identity element for multiplication:

2x1/2=1/2 + 1/2=1

Where as addition is a translation, multiplication is a magnifiation. Mathematics does not define a basic size, but fractions identifies the problem. Let us look at fractions, in base 4.



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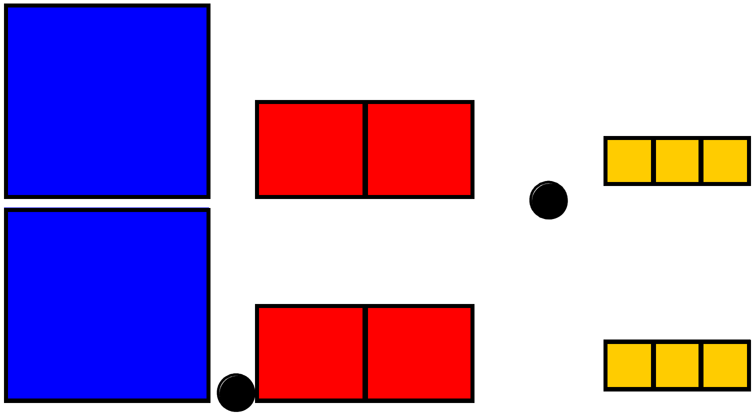
Fractions (continued)



If the blue square is one unit the red square is 1/4 of it, and the yellow square is 1/16 of the blue square. If the red square is one unit, the blue square is 4 units and the yellow square is 1/4 of the red square.

Thus 1.23= 1+2/4+3/16 = 1+11/16=27/16
And 12.3= 1*4+ 2x1+3/4=6+3/4=27/4

Since this was base four, we had to multiply by 4 to move the decimal point one position to the left. You can see why one has difficulty understanding fractions; partly due to the fact that the definition of length is arbitrary.

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Complex Numbers

Once we define a number as a vector, understanding complex numbers becomes much easier to do. It is through exponention that we discover complex numbers: 4^1/2=(2x2)^1/2=2

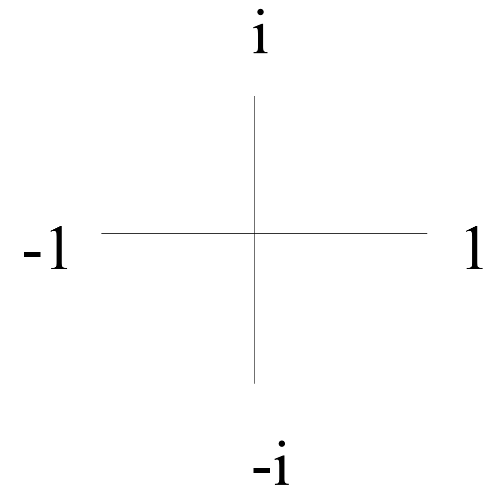
4^1/2=(-2x-2)^1/2=-2

(-4)^1/2==2 √-i   i^1= i

The we discover : i^2=i*i=-1

i^3=i*i*i=(i*i)*i=-i

i^4=(i*i)*(i*i)=-1x-1=1



We can see from the graph and the algebra, that every time we multiply by ‘i’, we rotate 90^0 (orthogonality). We also discover that e^iΦ=cos(Φ)+i*sin(Φ). Thus complex numbers introduce both rotation and dimension.

Summary

Kurt Gödel's work in logic, especially his Incompleteness Theorems, proved that any consistent axiomatic system powerful enough for basic arithmetic contains true statements that cannot be proven within that system.

The Turing Test, proposed by Alan Turing, is a benchmark for a machine's ability to exhibit intelligent behavior indistinguishable from a human's.

We showed that by using a vector model to interpret mathematics, we could explain the anomalies in mathematics, and then using the Turing approach to validate our inferences.

Math education can be changed by using this approach to show the fallacies in the existing approach, thus making the change seamlessly.