

Much Ado about Nothing

Introduction

In watching a video on math education, I began to realize I did not agree with what was being spoken, and wanted to challenge it without dismissing the topic altogether. In my own education I learned to do the following:

1. Explain what I was taught
2. Challenge what I was taught
3. Look for patterns
4. Explain the patterns
5. Apply my understanding in solving some problems
6. Solve the problems using a different approach
7. Challenge your explanation and understanding

Zero is nothing?

$a+0=a$ zero added to a number results in that number because 0 is nothing
 $0 \times a=0$ more about nothing

While I believe what was said, I think it could embarrass my thought process and could lead to errors in solving problems.

Is $2^0 = 0$ because exponentiation is multiple multiplication?
Is $0! = 0$ because factorials ($3!=3 \times 2 \times 1$) involve multiplication?

Actually,

$$2^0=1$$

$$0!=1$$

$$2^0=2^{1-1}=2/2=1$$

$$1!=1 \times 0! \quad 0!=1!/1=1/1=1$$

This pattern is interesting

**$A \times 1 = A$ 1 multiplied by number results in the number
because 1 is the identity element for multiplication**

$$1^A = 1$$

By symmetry leads me to:

$a + 0 = a$ 0 is the identity element for addition

**This sits much better with me. But could believing that $a + 0 = a$
and $0 \times a = 0$ because 0 is nothing cause me problems else
where?**

My Definition of Zero

Zero is the number that preceeds one (good but not quite right)

0	1	2	3	4
	0+1	1+1	2+1	3+1

Proof:

0+1=1 **creation of numbers**
3=1+2 **commutative Property**

3=(0+1)+2 **substitution, transitive property**

3=0+(1+2) **associative property**

3=0+3 **transitive property**

We have shown why zero being defined as the number before 1
Explains why $0+a=a$. We built upon the foundation of
mathematics-the commutative and associative properties

Problem with zero being nothing

$$3X2=2+2+2$$

$$2X2=2+2$$

$$1X2=2$$

$$0X2=?$$

$$3x2=0+2+2+2$$

$$2x2=0+2+2$$

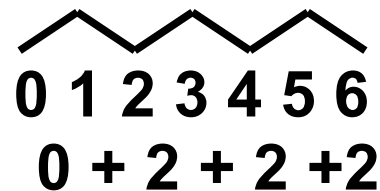
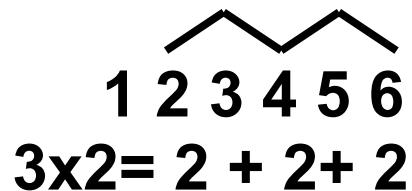
$$1x2=0+2$$

$$0x2=0$$

We kept dropping +2
In last step we dropped 2

We kept dropping +2
We illustrated that $0xa=0$

Skip counting



We had to redefine multiplication

The second diagram has been use to describe multiplication

Note how the jumps match the +s and note the start

Zero is a place holder?

Are we trying to justify that zero is a place holder?

Do we understand number structure?

$$4 \quad 4=4 \times 100 + 4=404$$

space holding place

$$404 = 4 \times 100 + 0 \times 10 + 4 \times 1 = 400 + 0 + 4$$

nicer and easier to present

$$414 = 4 \times 100 + 1 \times 10 + 4 \times 1 = 400 + 10 + 4$$

one is a place holder

Well, the one change the value

$$4.04 < 4.4 < 40.4$$

In this example zero changed the value

$$4.2 \quad |-2|4 = 4.2 \times 100 + -2 \times 10 + 4 = 420 - 20 + 4 = 404$$

In this case we did not need the zero

Here's a challenge

$$.001 = .1 - 9 - 9 = 1/10 - 9/100 - 9/1000 = (100 - 90 - 9)/1000 = 1/1000$$

While we have shown that we do not need the zero to make numbers, it makes computing with numbers much easier. And allows us to order numbers more easily,

We still need the zero to represent itself because 0 is the number before 1.

Does Zero have a sign?

$$\begin{array}{cccc} B & A & 0 & 1 \\ B+1 & A+1 & 0+1 & \end{array}$$

A is the number before 0 and B is the number before A

$$A+1=0$$

$$B+1=A$$

defintion

$$B+1+1=A+1$$

addition

$$B + 2 = 0$$

transitive

$$A=0-1$$

$$B=0-2$$

subtraction

$$A=-1$$

$$B=-2$$

notation

We have now shown how we named negative numbers

$$A+1=0$$

$$B+2=0$$

$$-1+1=0$$

$$-2+2=0 \quad \text{transitive}$$

We have just defined negative numbers

By using a different explanation, we made it easier to discover negative numbers
With better understanding

Conclusion

Should we stop teaching that zero is nothing, is a place holder, and has no sign? Mathematicians do not like to be shown that they do not understand mathematics. Mathematics has an evolutionary history of slowly accepting new approaches or concepts. Others, have probably observed parts of what I have shown.

However, by teaching both concepts together, we strengthen our understanding and confidence in mathematics. It also does not disrupt the way we teach math now, a major problem which occurs every time we try to replace what we have been taught previously.

You may notice that I have not tried to find errors in my approach. It turns out that the definition of zero is: Zero is the **integer** before 1 not the **number** before 1. I and my students discovered the problem in two ways: .5 is the number before 1 and in non-integer modular arithmetic, we have to scale the non-integers to integers (radians to degrees)