Smarter than your Math Teacher

I watched several youtube videos for finding the (Maclaurin) Series for 1/(1+X). Except for one, they all found the most difficult way to get half the answer. One did use the approach I am going to show you, but he too found half the answer.

We usually repeat what we are taught, often without asking whether there is another or better way or whether there is more to the problem.

The conventional way of finding 'the' infinite series of 1/(1+x) is to use differential calculus
However, there is a far easier way by just using elementary division.
However, after we find the answer, we find it only works for the values of $|x|<1$. This is where our math teacher usually stops.

There is another way, but you have to ask is there another way of doing the division. Now you will be able to find the values for $|X|>1$. This means that you need two series to represent 1/(1+x) over its full range.

But if you look closely at the series, you discover that they diverge for 1 and -1, but for
-1 it is ok. However, by representing the series by a geometric series formula, we can find that the series each approach .5 as x gets closer and closer to 1. This approach was used on one of the videos.

After looking at this lesson, ask yourself how you got smarter!

Here is how creativity works

```
    1 -x + x^2
1+x)1
    1+x
       -x
       -x-x^2
           x^2
           x^2+x^3
                -x^3
```

I can see that 1/(1+x) by pattern can be written as follows:

Y = 1/(1+x) = 1-x+x^2-x^3+x^4-...

If the division is to converge then x must be -1<x<1 since x^n approaches zero as n becomes larger and larger

The formula for the sum of a geometric progression is derived as follows;

S= a+ab+ab^2+ …. + ab^n

bS= ab +ab^2+ + ab^n+ab^(n+1)

S-bS = a-ab^(n+1) only first and end term left!

S(1-b) = a(1- b^(n+1)) distributive property

S= a(1- b^(n+1)) / (1-b)

For y=1/(1+x) b=-x a=1 giving us (1-(-x)^(n+1))/1+x) if -1<x<1 and n goes to infinity, then the sum becomes 1/(1+x). Put in a few value of x to check it out.

X=.5 y=1/(1+1/2)=2/3=.66666

Y=1-.5+.25-.125+0625-.03125-.=.66625 i.e. sum gets closer and closer to 2/3

Can we make the ratio work for x<-1 and x>1?

```
        1/x-(1/x)^2+ (1/x)^3-
x+1)1
         1+1/x
            -1/x
             -1/x-(1/x)^2
                      (1/x)^2
```

I can see that 1/(x+1) by pattern can be written as follows:
Y=1/(x+1)=1/x-(1/x)^2+(1/x)^3-(1/x)^4+...

If the division is to converge then x must be x<-1 or x>1 since (1/x)^n approaches zero as n becomes larger and larger.

For y=1/(x+1) b=-1/x a=1/x giving us (1/x)(1-(-1/x)^(n+1))/1+1/x) if x<-1 or x>1 and n goes to infinity, then the sum becomes 1/(x+1). Put in a few values of x to check it out.

X=9 y=1/(9+1)=1/10=.100000....
Y=1/9-1/81+1/729..=.111111...-.012345....+.0013717=.100137

Conclusion

$Y=1/(1+x)$ is represented by two infinite series

$-1<x<1$ Small Number Limit
$Y=1/(1+x)=1-x+x^2-x^3+x^4-\ldots$

$X<-1$ $x>1$ Either very large negative or very large positive limit
$Y=1/(1+x)=1/x-(1/x)^2+(1/x)^3-(1/x)^4+\ldots$

For $x=1$ we see that as x approaches 1 the series very slowly converges to .5 We take .5 as the value for $x=1$.

For $x=-1$ the series diverges to infinity (so does $1/(1+x)$).

We have discovered that we need two series to define or evaluate it. We discovered a way to get around the divergence at x exactly equal 1.

I searched the youtubes that found the series for $1/(1+x)$ or $1/(1-x)$
All of them found a more difficult way to find the first series. None bothered to find the second series:

https://www.youtube.com/watch?v=Ux5hvvceZis
https://www.youtube.com/watch?v=Ux7vl6zXxj0
https://www.youtube.com/watch?v=y63FprQC_po
https://www.youtube.com/watch?v=7t19BeWnrMc

Solving an anomaly

There are two ways to divide 1+x into 1.

$|x|<1$

$$
\begin{array}{r|l}
 & 1-x+x^2-x^3+x^4-\ldots \\
\hline
1+x & 1 \\
 & \underline{1+x} \\
 & -x \\
 & \underline{-x-x^2} \\
 & x^2 \\
 & \underline{x^2+x^3} \\
 & -x^3 \\
 & \underline{-x^3-x^4} \\
 & x^4
\end{array}
$$

$|x|>1$

$$
\begin{array}{r|l}
 & x^{-1}-x^{-2}+x^{-3}-x^{-4}+\ldots \\
\hline
x+1 & 1 \\
 & \underline{1+x^{-1}} \\
 & -x^{-1} \\
 & \underline{-x^{-1}-x^{-2}} \\
 & x^{-2} \\
 & \underline{x^{-2}+x^{-3}} \\
 & -x^{-3} \\
 & \underline{-x^{-3}-x^{-4}} \\
 & x^{-4}
\end{array}
$$

A lot of fuss is made over that the series on the left does not represent 1/(1+x) for values of x greater or equal to 1. We show that by introducing another series that represents 1/(1+X), that the series does represent 1/(1+x) for values of x greater than 1.

Solving an anomaly

There are two ways to divide 1+x into 1.

$|x|<1$

$$\begin{array}{r} 1-x+x^2-x^3+x^4-\ldots \\ 1+x\overline{)1} \\ \underline{1+x} \\ -x \\ \underline{-x-x^2} \\ x^2 \\ \underline{x^2+x^3} \\ -x^3 \\ \underline{-x^3-x^4} \\ x^4 \end{array}$$

Pick X = 0.1

$$\begin{array}{r} 1-.1+.1^2-.1^3+.1^4-\ldots \\ 1+.1\overline{)1} \\ \underline{1+.1} \\ -.1 \\ \underline{-.1-.1^2} \\ .1^2 \\ \underline{.1^2+.1^3} \\ -.1^3 \\ \underline{-.1^3-.1^4} \\ .1^4 \end{array}$$

A lot of fuss is made over that the series on the left does not represent 1/(1+x) for values of x greater or equal to 1. We show that by introducing another series that represents 1/(1+X), that the series does represent 1/(1+x) for values of x greater than 1.

Solving an anomaly

There are two ways to divide 1+x into 1.

Pick x=10

$$10^{-1}-10^{-2}+10^{-3}-10^{-4}+\ldots$$

$$10+1\overline{)1}$$

$$\underline{1+10^{-1}}$$

$$-10^{-1}$$

$$\underline{-10^{-1}-10^{-2}}$$

$$10^{-2}$$

$$\underline{10^{-2}+10^{-3}}$$

$$-10^{-3}$$

$$\underline{-10^{-3}-10^{-4}}$$

$$10^{-4}$$

$|x|>1$

$$x^{-1}-x^{-2}+x^{-3}-x^{-4}+\ldots$$

$$x+1\overline{)1}$$

$$\underline{1+x^{-1}}$$

$$-x^{-1}$$

$$\underline{-x^{-1}-x^{-2}}$$

$$x^{-2}$$

$$\underline{x^{-2}+x^{-3}}$$

$$-x^{-3}$$

$$\underline{-x^{-3}-x^{-4}}$$

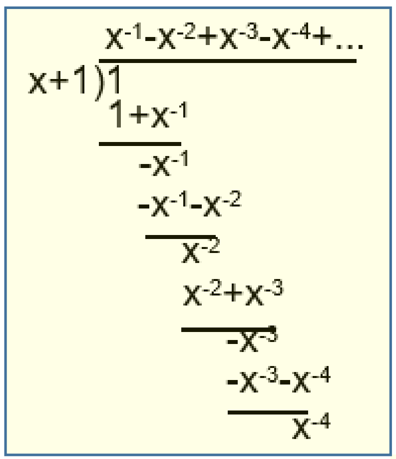
$$x^{-4}$$

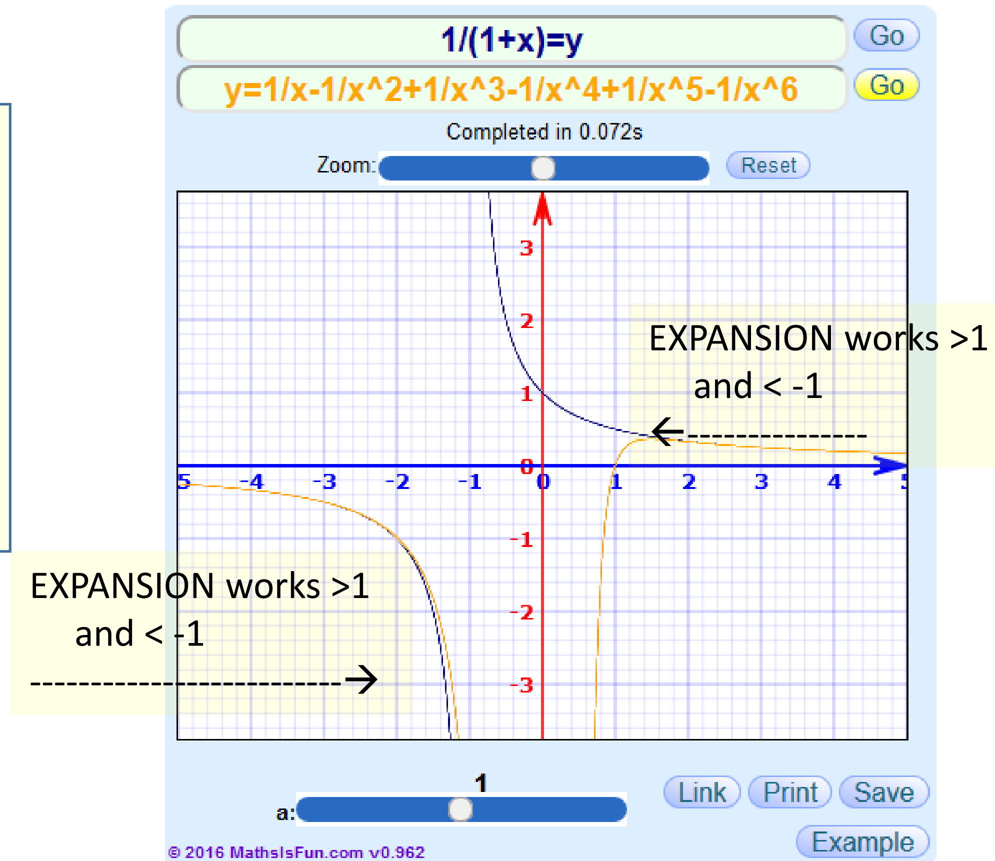
A lot of fuss is made over that the series on the left does not represent 1/(1+x) for values of x greater or equal to 1. We show that by introducing another series that represents 1/(1+X), that the series does represent 1/(1+x) for values of x greater than 1.

1/(1+x) vs. expansion 1/x-1/x^2+1/x^3

First factoring order x+1 into 1 gives 1/x-1/x^2+1/x^3 expression – good at wings

Works from 1.5 to infinity; also -1.5 to negative infinity

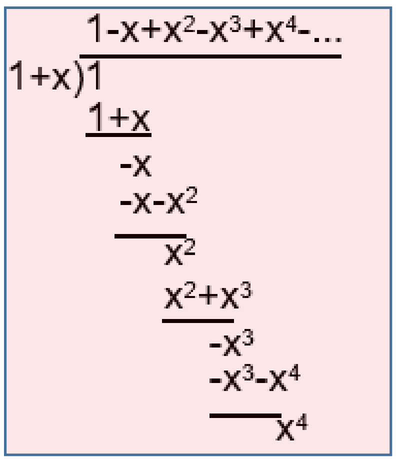


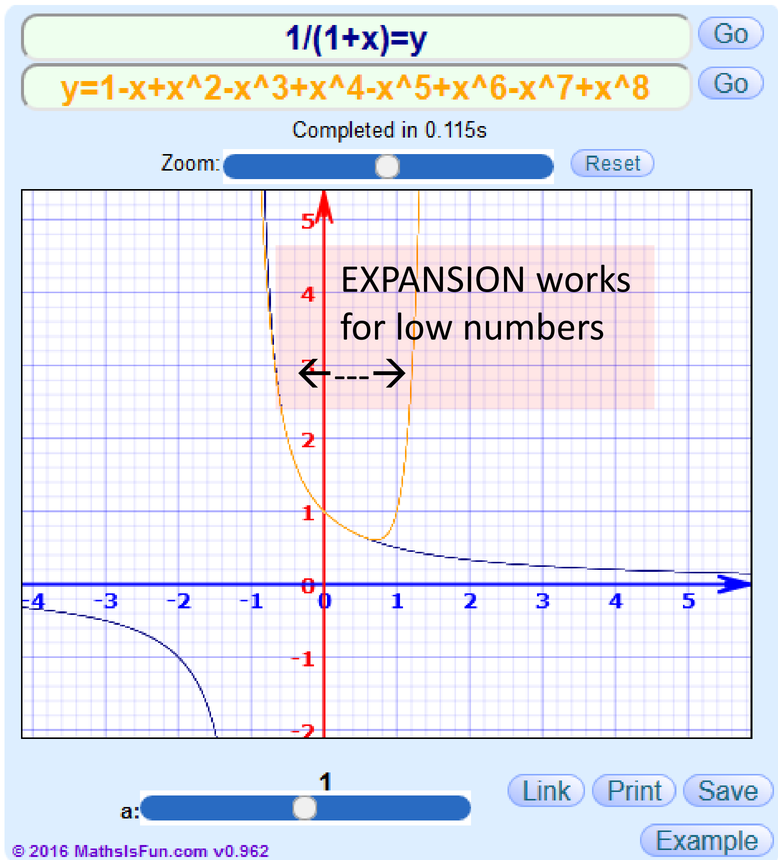


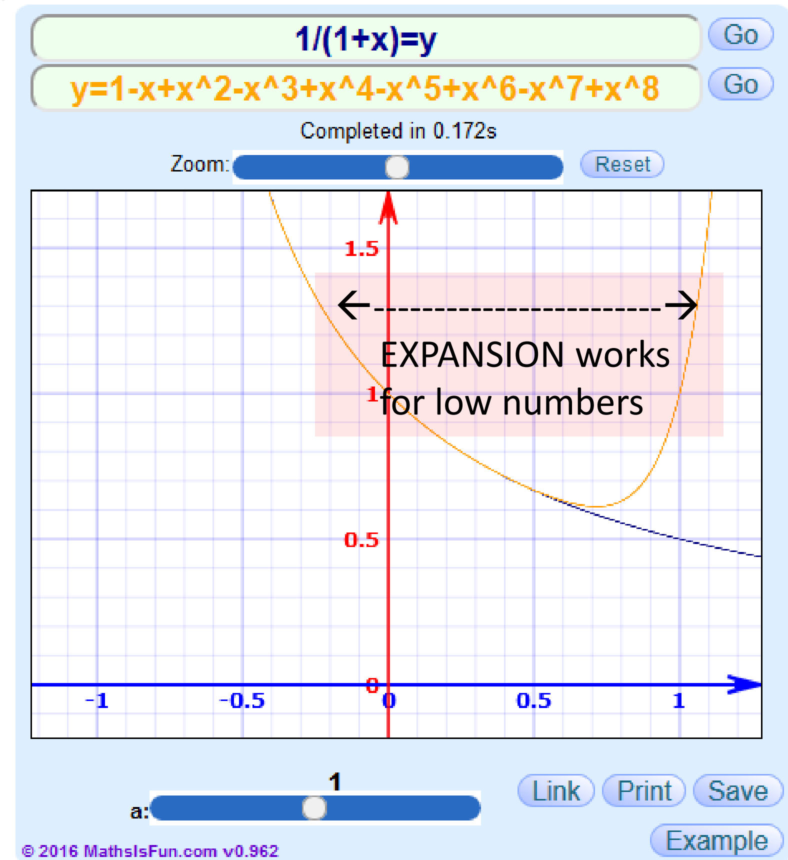
Credit and thanks for Graphics Program used here:
https://www.mathsisfun.com/data/grapher-equation.html

1/(1+x) vs. expansion 1-x+x^2-x^3+x^4 …

Alternate Factoring order 1+x into 1 gives 1-x+x^2-x^3 expression - good near origin
Works from -0.9 to 0.7, doesn't catch high end or branch less than -1







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How to calculate the infinite series

For a geometric series $s=a+ab+ab^2+...ab^{n-1}=a(b^n-1)/(b-1)$

$s=a+ab+......+ab^{n-2}+ab^{n-1}$ multiply by b

$bs= ab+... +ab^{n-1}+ab^n$

$(1-b)s=a-ab^n$

$s=a(1-b^n)/(1-b)$ multiply by -1/-1 $s=a(b^n-1)/(b-1)$

Checking our work

s= 1+2+2^2+2^3=1+2+4+8=15

$s=1*(2^4-1)/(2-1)=(16-1)/1=15$

s=1/2 + 1/4 +1/8 +1/16=15/16

$s=1/2(1/2)^4-1)/(1/2-1)=(1-1/16)=15/16$

Giving ourselves confidence

a=1 b=-x $s=1 *((-x)^n-1)/(-x-1)$

$=(1-(-x)^n)/(x+1)$

As n>∞ =1/(1+x)

$a=x^{-1}$ $b=-x^{-1}$

$s=x^{-1}((-x)^{-n}-1)/(-x^{-1}-1)$ mult. by x/x

$=(1-(-1/x)^n)/(1+x)$

=1/(1+x)

They come out the same

Further Analysis

1/(1+x) approaches infinity as x approaches -1.

Both series diverge

$\|x\|<1$	$\|x\|>1$
$s=(1-(-x)^n)/(x+1)$	$s=(1-(-1/x)^n)/(1+x)$

If x=1

$s=(1-(-1)^n)/2$ $s=(1-(-1)^n/2$

The sum fluctulates between 0 and 1 depending upon whethe n is odd or even,

However if x approaches 1, the series will converge to ½, but very slowly.

Note that $.9999^{\infty}=0$