Fourth Dimension

Explaining Complex Roots

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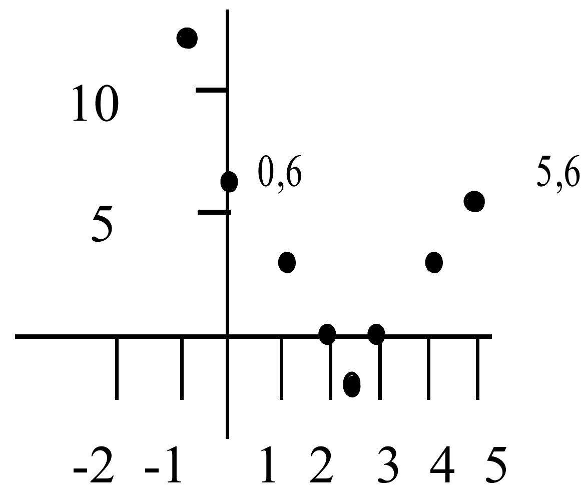
Roots of a polynomial.

The problem starts with what we mean by roots of a polynomial.

Y=(x-2)*(x-3)=x^2-5x+6. The roots of that polynomial are 2 and 3 because it is at these points that the curve crosses the x=0 axis. Let us build a table and plot a graph.

y= x^2-5x+6

x	y
-1	12
0	6
1	2
2	0
.5	.25
3	0
4	2
5	6



Let us now consider another function.

y = (x-(1+i))*(x-(1-i)) = **x^2-2x+2**

y = x^2-2x+2

x	y
-2	10
-1	5
0	2
1	1
2	2

x	-2	-1	0	1	2
2i	6-12i	1+8i	-2-4i	-3+0i	-2+4i
1i	9-6i	4-4i	1-2i	**0+0i**	1+2i
0i	10+0i	5+0i	2+0i	1+0i	2+0i
-1i	9+6i	4+4i	1+2i	**0+0i**	1-2i
-2i	6+12i	1-8i	-2+4i	-3+0i	-2-4i

First we look at substituting real values in the equation. This gives us real values for "y" which match the middle blue row in the second table. The entries in the second table are result of putting a complex value for x. The top row represents the real component and the vertical row represents the imaginary component of y . Thus, f(2+2i)=-2+4i. We also see that there are only two places where both the real part and the imaginary parts are both 0. This is the plain where y=0 intersects the curves.

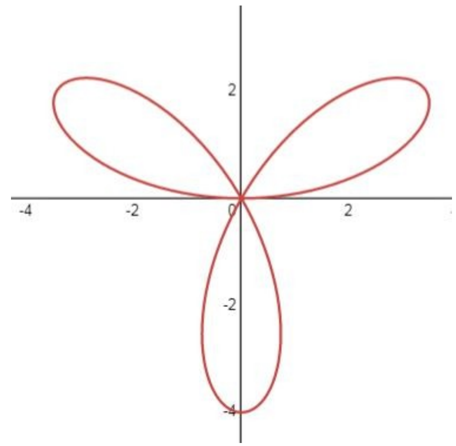
	y					yi				
x	-2	-1	0	1	2	-2	-1	0	1	2
2i	6	1	-2	-3	-2	-12i	8i	-4i	0i	4i
1i	9	4	1	**0**	1	-6i	-4i	-2i	0i	2i
0i	10	5	2	1	2	0i	0i	0i	0i	0i
-1i	9	4	1	**0**	1	6i	4i	2i	0i	-2i
-2i	6	1	-2	-3	-2	12i	-8i	4i	0i	-4i

In these two tables we can look at the vertical rows (real plains) and the horizontal rows (imaginary plane) to see a parabolic cross section. This is easier to see if we do this in conjunction with contour plots.

This allows us to interpret the complex number as a vector. We are familiar with the three spatial dimensions, We now see six. I have not yet figured out what this does to the dimension of time. Perhaps, there is a way to go faster than the speed of light. On the series ‘Star Trek’, they do reference warp speed (faster than the speed of light). Anti matter was theorized by Paul Dirac through the use of ‘imaginary’ numbers.

Calculus

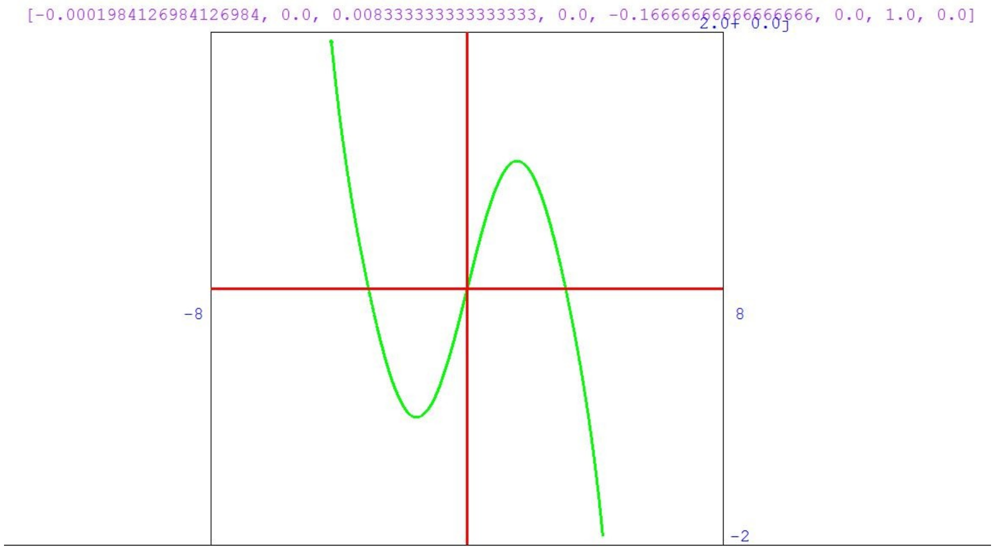
Besides introducing rotation and dimension, exponentiation introduces irrational numbers — numbers that can not be represented by fractions. This also leads us to the concept of continuity. Look at the curve for 4sin(3ø):



A curve is initially plotted with a few points to give us a feel for the shape of the curve. As we add more and more points the curve takes on a more solid look. We then see crossings at the origin. We can see three arcs intersecting at the origin. To distinguish these arcs we use calculus. The slope of the arc going into the intersection should match that of the arc coming out of the intersection. If they do not match, we consider the curve discontinuous at that point. This simple interpretation brings in a new area or concept of math. We also see how the topics of angles and continuity begin to be ordered.

Let us look at how a problem evolves. We start off checking the factorization of the infinite series for the sine to evaluate the value of pi. First we look at the equation:

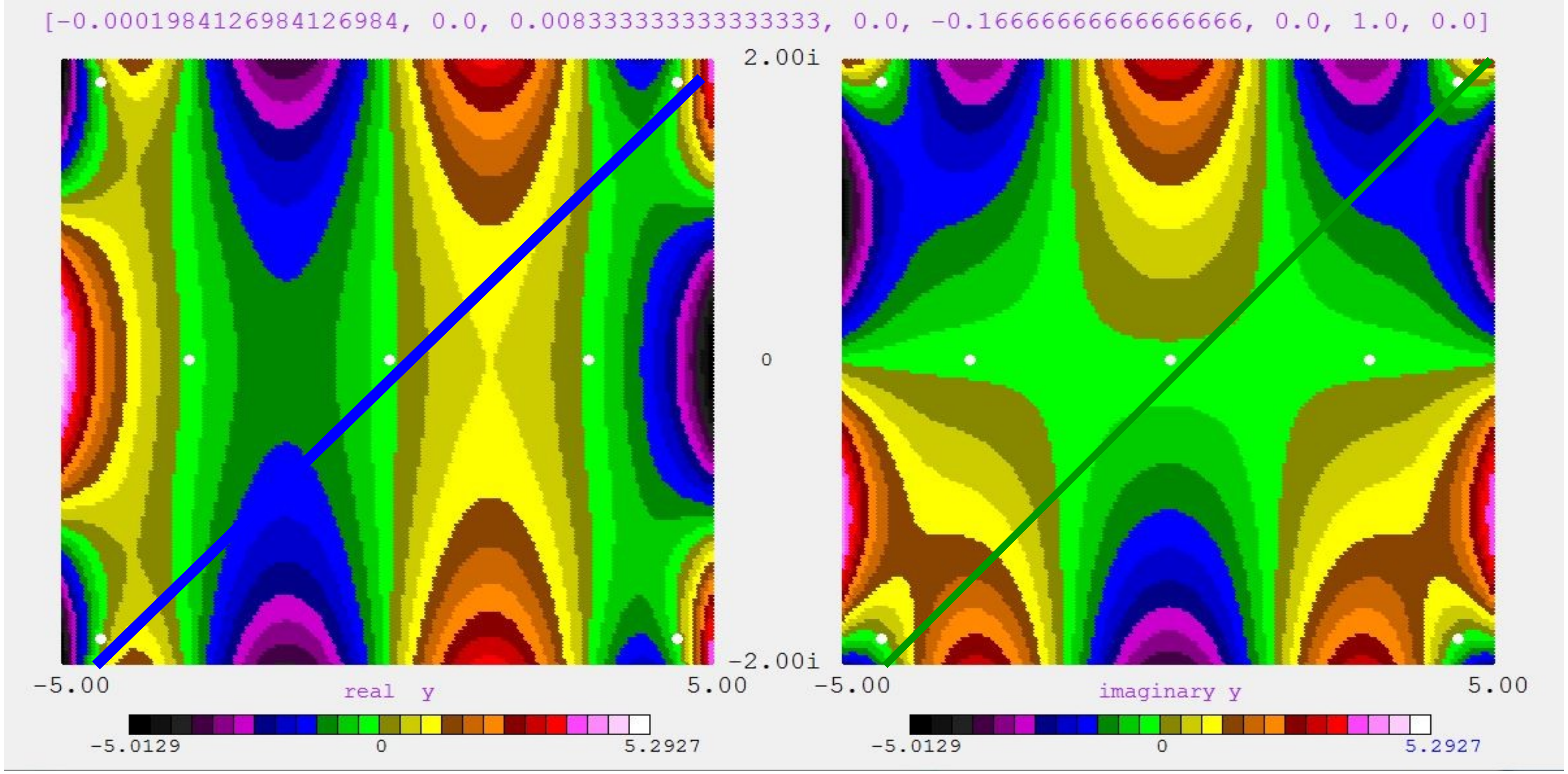
$$y=-x^7/7!+x^5/5!-x^3/3!+x\backslash 1!$$

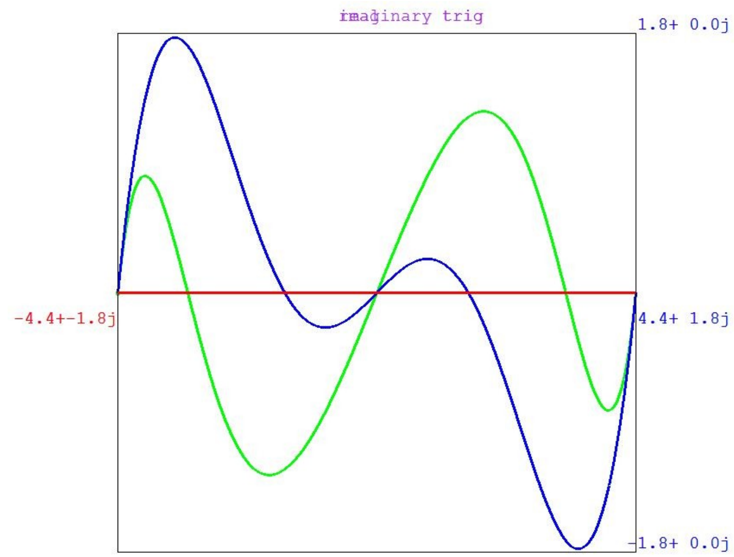


This is because we are only looking at a slice of the curve in the x-y plane. To see the rest of the crossings, we have to look in the x-xi plane. This becomes a four dimension plot in which we can only look at three of those dimensions through a contour plot.

Here we are using colors to express the numeric values. The khaki green color is where the zeros are in the curve. We have drawn a gray line through two complex points (-4.434005-1.843752j , 4.434005+1.843752j) and one real point (0,0).

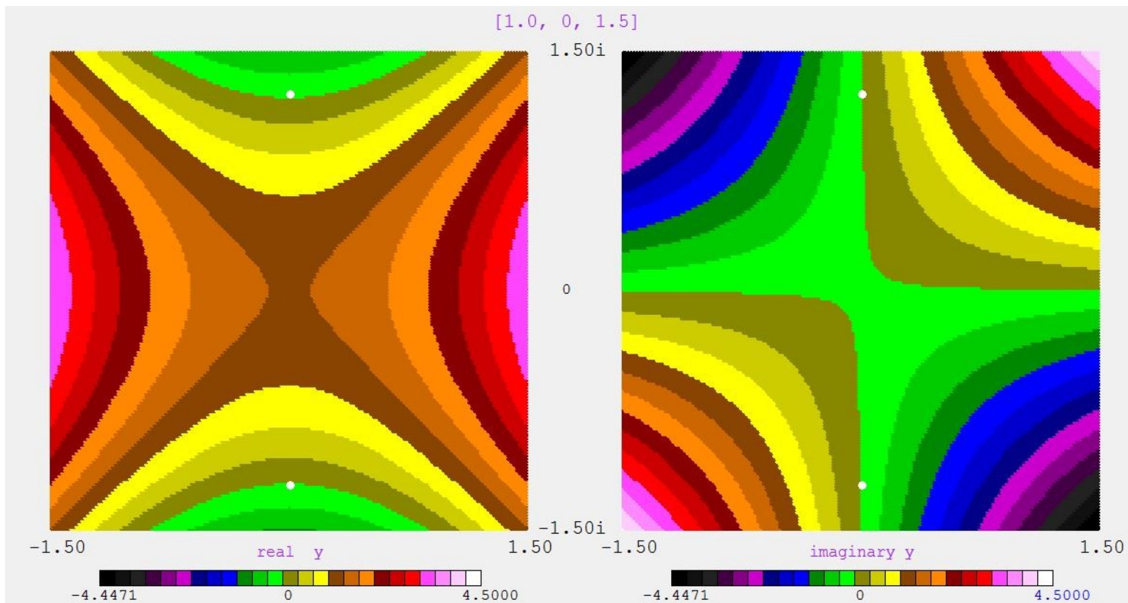
If we draw a line through those three points, we get the following graphs where the blue curve is the real part of y and the green curve is the imaginary part of y.

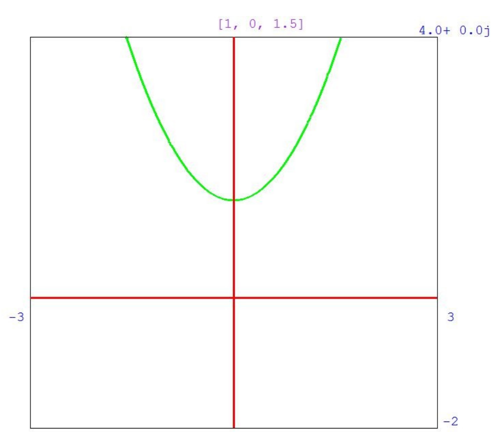


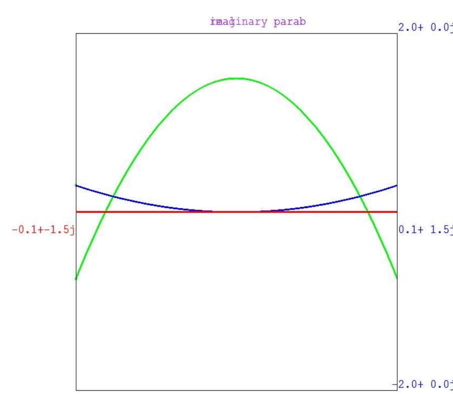


While both curves cross the zero line several times, they only cross it simultaneously, three times. Thus both the real and imaginary parts of must be zero simultaneously to be considered a root of the equation. It is because we have not used all of the presentation tools available to us that we could not comprehend what a complex zero crossing meant. Standard plotting packages such as Demos and Mathsisfun do not handle complex numbers. We needed the contour plot or a 3d printer and the graphing program to fully understand what is happening, We begin to realize that complex numbers constitute another dimension.

Let us look at the graph for a parabola with which everyone is familiar. The top right diagram shows the contours of the real part of y. We can see a cross section of it along the real x axis. In the graph below the contours. The graph to the right of that shows y snd yi plot with a cross section through the imaginary roots. The blue line representing yi is slightly of from the zero cut so we could understand the requirement that both yi and y be zero at the same point. The figure on the bottom left is a 3D model of that contour plot.







Summary

We used the Newton-Rapheson formula starting with a complex seed to find the roots of these equations. The Python program for complex and multiple roots. Python programs were also written to solve for these roots. We had to address the limitations of precision to get accurate results. A better choice of colors could have been made to measure the heights of the contour plots.

We used a three d printer so that we could have a more tangible feel for what we wer doing.

As w got better understanding of complex roots, we realized that we we were working ith four dimensions: x, xi, y, yi